Int. J. Heat Mass Transfer. Vol. 16, pp. 1944–1949. Pergamon Press 1973. Printed in Great Britain

LAMINAR FILM CONDENSATION WITH VAPOUR DRAG ON A FLAT SURFACE

Y. R. MAYHEW and J. K. AGGARWAL*
Mechanical Engineering Department, University of Bristol, Bristol, England

(*Received* 28 *September* 1972 *and in revised form* 4 *April* 1973)

NOMENCLATURE

- Dr, dimensionless friction drag number ($= \rho\tau_\delta h'_{fg}l^2/\mu k\theta_s$);
- D, diameter of the porous tube;
- d_e, equivalent diameter of rectangular condensation duct ($= 4 \times$ cross sectional area/perimeter);
- F, shear force at liquid–vapour interface;
- f, friction factor under suction conditions;
- f_0, friction factor in the absence of suction;
- g, gravitational acceleration;
- h, h_l, average condensation heat transfer coefficient between $x = 0$ and $x = l$; terminal condensation heat transfer coefficient at $x = l$;
- h'_{fg}, liquid–vapour enthalpy change modified for liquid film undercooling ($= h_{fg} + \frac{3}{8}c_p\theta_s$);
- k, thermal conductivity of liquid;
- L, length of the porous tube;
- l, total length of the condensation test section;
- m, condensate mass flow rate per unit width at section x;
- Nu, Nu_l, average Nusselt number ($= hl/k$); terminal Nusselt number ($= h_l l/k$);
- Re_D, Reynolds number of air in suction experiments ($= \rho\bar{u}D/\mu$);
- Re_v, 'two-phase' Reynolds number ($\rho\bar{u}_v l/\mu$);
- Sh, the condensation group $\rho^2 g \sin\phi h'_{fg}l^3/4\mu k\theta_s$;
- t_s, t_w, saturation temperature, wall temperature;
- $\bar{u}$, local weighted mean air velocity along tube in suction experiments;
- $\bar{u}_v$, weighted mean vapour velocity in condensation duct;
- u_δ, velocity at liquid–vapour interface;
- v_w, suction velocity of air normal to, and at, the porous wall;
- x, axial distance from beginning of condensation section, or from entry of porous tube;
- β, suction coefficient ($= v_w/\bar{u}$);
- δ, δ_l, liquid film thickness at x, at $x = l$;
- θ_s, the temperature difference ($t_s - t_w$);
- μ, dynamic viscosity of liquid in condensation experiments, or of air in suction experiments;
- ρ, density of liquid in condensation experiments, or of air in suction experiments;
- ρ_v, density of vapour in condensation duct;
- τ_w, shear stress with suction at porous tube wall ($=\frac{1}{2}f\rho\bar{u}^2$);
- τ_0, wall shear stress without suction ($=\frac{1}{2}f_0\rho\bar{u}^2$);
- τ_δ, 'Blasius' shear stress at liquid–vapour interface;
- ϕ, inclination of the condensation surface to the horizontal;
- ψ, momentum loss coefficient defined by equation (2).

* Now at Engineering Science Department, University of Oxford.

1. INTRODUCTION

FROM condensation experiments carried out in a short vertical tube (25 mm dia × 200 mm long) it was found that the observed rates of condensation were much larger than those predicted by Nusselt's theory when using a value for the vapour drag calculated from a Blasius-type law. To resolve the discrepancy, a theory was developed by Mayhew *et al.* [1] which accounted satisfactorily for the observed results, and this theory was discussed further in [2].

To extend the range of the parameters of the original investigation, a new apparatus incorporating a flat condensation surface, forming one side of a rectangular duct, has been built. The advantages of this design were several: (1) Being able to place the cooled surface horizontally, the gravity force on the film could be eliminated and the results be made more sensitive to the effect of vapour drag; (2) It was possible to carry out experiments with counterflow, with the surface vertical and the vapour flow upwards; (3) Observations through a window placed in the working duct wall opposite the condensation surface made it possible to check for absence of drop-wise, rippled and turbulent-film condensation.

The theory outlined in [1] assumed that the value of the shear force F at the liquid–vapour interface of length dx and of unit width (or unit periphery for a tube) could be expressed as a sum of two terms, viz.

$$F = \tau_\delta \, dx + (\bar{u}_v - u_\delta)\, dm \approx \tau_\delta \, dx + \bar{u}_v \, dm \quad (1)$$

where the stress τ_δ was a value calculated from a Blasius-type law, and dm was the condensation rate over the same area. The second term represented a momentum transfer to the interface equivalent to a reduction of momentum of the condensing vapour from its value $\bar{u}_v \, dm$ to the usually negligible interface value $u_\delta \, dm$. Clearly, there exists only a single total shear force F at the interface, and the superposition of two hypothetical forces is merely a device to predict its total value.

There is, however, no reason to believe that the momentum transfer to the interface should be exactly equal to $\bar{u}_v \, dm$, and it will be influenced by the conditions of flow prevailing locally in the region of the interface. It is therefore proposed that equation (1) should be modified to read

$$F = \tau_\delta \, dx + \psi \bar{u}_v \, dm \quad (2)$$

where ψ will be called the momentum loss coefficient. It is difficult to see how ψ could be predicted theoretically although one would expect ψ to have a value less than unity. [Beckett and Poots have shown in [3], that when both vapour and film flows are laminar and condensation rates are high, equation (2) can be written as $F = \psi \bar{u}_v \, dm$, where $0{\cdot}94 \leqslant \psi \leqslant 1$, see their equation (4.11).] It is possible, however, to compare values of shear force deduced from condensation experiments with those from suction experiments in which air flows past a porous surface. Such a flow model represents a simple hydrodynamic analogue of a condensing vapour flowing past its own condensate film. Agreement would provide firm support for this simple modification of the theory.

2. MODIFIED VERSION OF PREVIOUS THEORY

With the assumptions about the shear stress outlined in the Introduction, it was shown in [1], implicitly assuming $\psi = 1$, that the film thickness δ_l at the end of the cooled surface of length l is given by

$$(Sh)\left(\frac{\delta_l}{l}\right)^4 + \tfrac{1}{3}(Dr)\left(\frac{\delta_l}{l}\right)^3 + \tfrac{1}{4}(Re_v)\left(\frac{\delta_l}{l}\right)^2 - 1 = 0. \quad (3)$$

The local Nusselt number Nu_l at that point was shown to be

$$Nu_l = \frac{l}{\delta_l} \quad (4)$$

and the average Nusselt number Nu up to $x = l$ to be

$$Nu = \tfrac{4}{3}(Sh)\left(\frac{\delta_l}{l}\right)^3 + \tfrac{1}{2}(Dr)\left(\frac{\delta_l}{l}\right)^2 + \tfrac{1}{2}(Re_v)\left(\frac{\delta_l}{l}\right). \quad (5)$$

For the horizontal plate the term including Sh becomes zero because $g \sin \phi = 0$. Dr is a dimensionless parameter arising from the τ_δ term in equation (1), while the Re_v parameter arises from the $\bar{u}_v \, dm$ term.

If it is assumed as a first approximation that ψ = constant, i.e. ψ is independent of x, the derivation outlined in [1] can be retraced to yield equations identical with equations (3), (4) and (5), except that the terms containing Re_v are multiplied by the assumed value of ψ.

To assess the importance of the 'friction' (Dr) term compared with that of the 'momentum' (Re_v) term, each term in turn can be ignored in the evaluation of the Nusselt number (Nu). Theoretical curves involving various combinations of terms included in the evaluation of Nu are superimposed on some of the graphs of experimental results presented here. It will be seen later that the effect of the Dr term on the prediction of Nu is much less than that of the Re_v term for the conditions encountered in the experiments.

3. CONDENSATION EXPERIMENTS AND DISCUSSION OF RESULTS

The test section consisted of a rectangular duct about 100 mm × 25 mm in cross section and 150 mm long. A 600 mm long entry duct ensured that the vapour arrived in fully-developed turbulent flow. The cooled condensation surface, forming one side of the test section, was divided into a central strip of 50 mm width, with a guard strip on each side.

The range of Re_v covered with the surface vertical, inclined and horizontal corresponded to vapour velocities from about 6 m/s to 60 m/s, all with co-current flow. Experiments with counterflow up to 7 m/s were also conducted with the surface vertical.

Figures 1 and 2 show the results obtained at extreme values of ϕ. Generally speaking, the theory can predict condensation rates satisfactorily, and a good approximation is provided even when the Dr term is neglected. The following reservations must, however, be made.

Firstly, with the surface horizontal (Fig. 1), the results at low vapour velocity are much higher than those predicted. Indeed this must be so because the theory for a horizontal surface, in the limiting case of $Re_v \to 0$, predicts $\delta \to \infty$ and $Nu \to 0$, and it does not allow for the drainage of the film away from the edges of the finite surface used in the experiments.

Secondly, in counterflow with the surface vertical (Fig. 2, Re_v negative), the results are appreciably higher than those predicted, and indeed always higher than the corresponding no-drag ($Re_v = 0$) value. An obvious explanation was provided by dye-injection tests which showed that, with counterflow, no laminar film flow could be achieved. The film was torn off the plate (i.e. flooding occurred) at quite moderate values of vapour velocity.

Similar observations with parallel flow confirmed that the film was always both laminar and smooth. From work with non-condensing films it was expected that rippled flow would be encountered over part of the surface at the higher velocities used. In fact remarkably smooth films were observed,

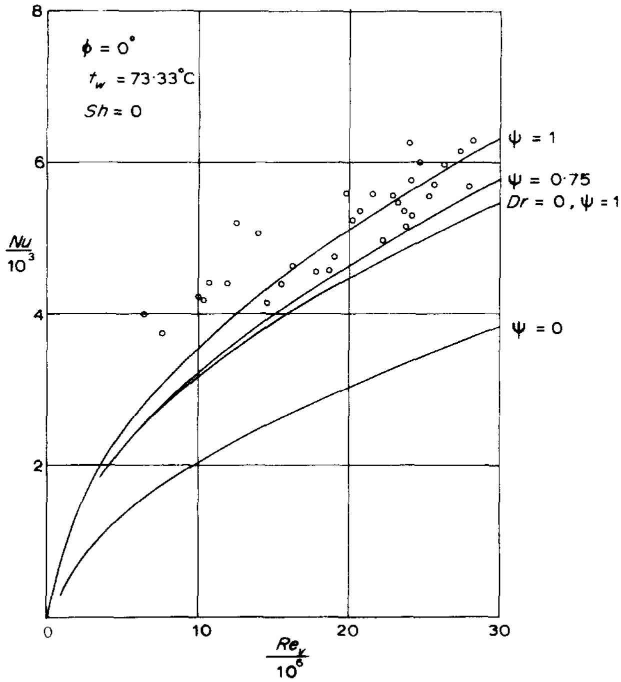

FIG. 1. Condensation on a horizontal plate with horizontal steam flow. (N.B. The scatter of experimental results in Figs. 1 and 2 is in part due to the fact that these results were obtained with various wall temperatures t_w, lying mainly between 70°C and 78°C. The theoretical curves were computed for a nominal value of t_w = 73·33°C. It was estimated that the variation of t_w in the range 70–78°C would account for a scatter band in *Nu* of about 200.)

suggesting that mass transfer, and possibly also surface tension effects on the non-isothermal film, must have had a stabilizing effect.

4. COMPLEMENTARY EXPERIMENTAL WORK WITH WALL SUCTION

The interfacial shear stress in the analogous situation of gas flow with wall suction has been measured using a sintered-bronze tube (Porosint, L/D = 9·3). The scope of the work reported in [4] and [5] was considerably wider than indicated here, and only a few immediately relevant results have been extracted and reanalysed for this communication. Basically, the experimental apparatus consisted of a porous tube fed with air in fully-developed turbulent flow and subjected to uniform surface suction. The proportion of entry air extracted through the wall was varied from 0 to 100 per cent, and the axial gradients of static pressure and radial distributions of axial velocity were measured.

The determination of wall shear stress from these experiments, as opposed to that from condensation experiments, has its own difficulties. The stress was deduced from measurements of axial pressure changes, and these changes are affected not merely by the wall shear stress, but also by the 'diffusion' effect resulting from suction, the net effect usually being a pressure rise in the flow direction. Thus the calculation of wall shear stress depends upon the difference of two quantities of often similar order of magnitude, and careful experimental work and a sophisticated evaluation of the data is necessary to achieve the required accuracy (see [4]).

By contrast, in the condensation experiments the value of the shear stress was deduced from the value of the heat transfer; that the latter was known accurately was proved by good energy balances between the condensate and cooling water. The pressure variation along the duct only affected the saturation temperature t_s and the enthalpy change h'_{fg}, and uncertainty arising from this was very small because of the short test section and high by-pass flow used. The greatest likely source of error, as in all such experiments, lies in the measurement of the wall temperature t_w, and hence in the

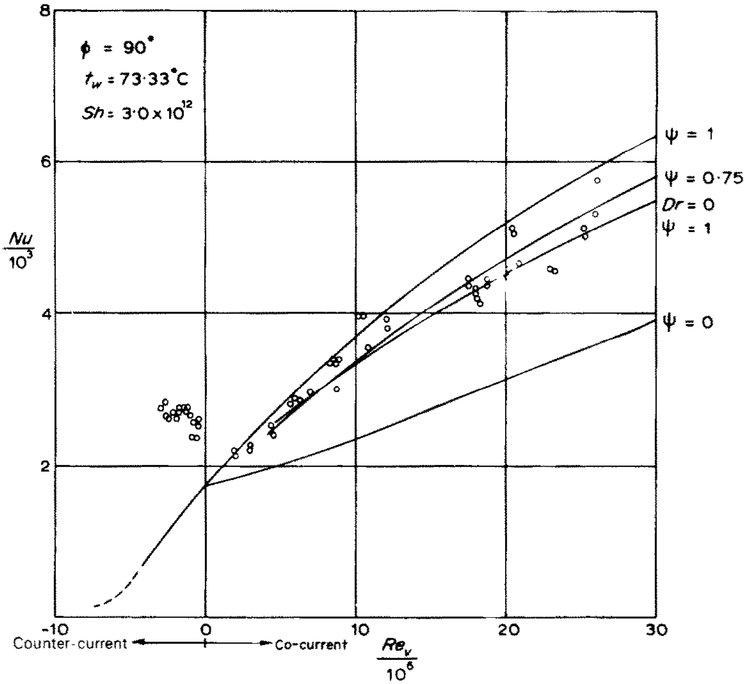

FIG. 2. Condensation on a vertical plate with counter-current and co-current steam flow.

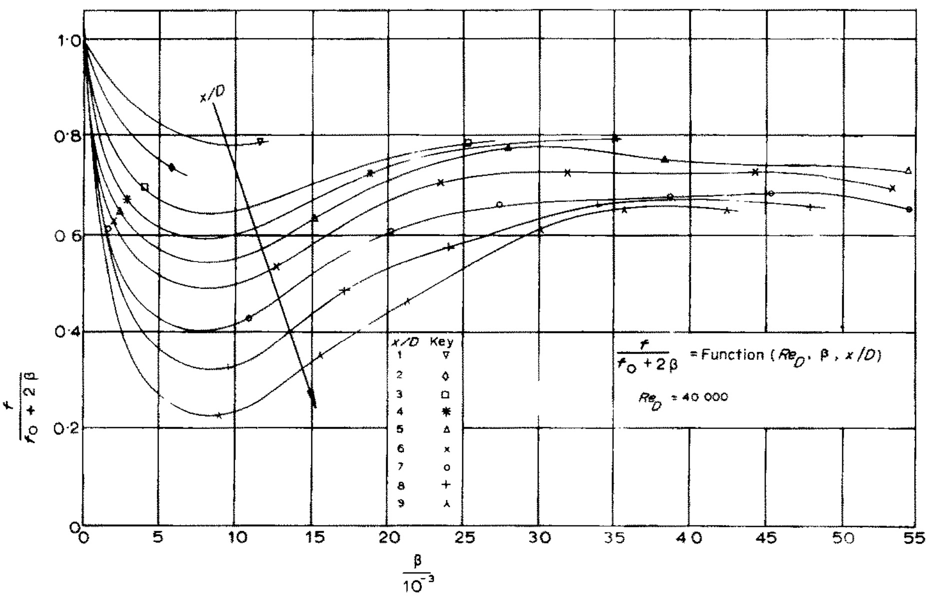

FIG. 3. Local non-dimensionless shear stress for Porosint tube at Re_D = 40000.

value of θ_s needed to calculate the heat transfer coefficient h from the value of the heat transfer.

The local wall shear stress τ_w deduced from the suction experiments has been normalized by the term $\tau_0 + \rho\bar{u}v_w$, where $\tau_0 = \frac{1}{2}f_0\rho\bar{u}^2$ is the shear stress in the absence of suction, ρv_w is the mass extraction rate per unit area of the wall, and $\rho\bar{u}v_w$ is its associated 'free-stream' momentum. The resulting normalized friction factor, viz. $f/(f_0 + 2\beta)$, has been plotted against the local suction parameter β, defined as the ratio $v_w/\bar{u}$. Figure 3 shows a typical set of results, relating to a fixed value of $Re_D = 40000$ at the sections at which τ_w was evaluated.

It is readily seen that, in terms of the nomenclature used for the suction results, equation (2) can be expressed as

$$\tau_w = \tau_0 + \psi\rho\bar{u}v_w \quad (6)$$

so that the momentum loss coefficient ψ is given by

$$\psi = \frac{f - f_0}{2\beta}. \quad (7)$$

Values of ψ, corresponding to the results of Fig. 3, are depicted in Fig 4.

Examination of all the results obtained so far (at other values of Re_D and with another type of porous tube) indicate that at high values of β, values of ψ lie in the range 0·6–0·9, averaging at about 0·75. Values in the range of 0·5–1·0 have previously been suggested explicitly or implicitly by several authors, e.g. see [6]. ψ appears to depend not only on β, but also on x/D and Re_D; preliminary results with a woven steel tube also suggests that it depends on the surface structure of the wall. It is at present impossible to suggest closer rules about how to estimate ψ, and considerable further work is necessary before such rules can be formulated.

At high suction rates τ_0 becomes small in comparison with $\rho\bar{u}v_w$, and it is therefore to be expected that the normalised shear stress and ψ should become approximately equal. This is indeed indicated by a comparison of Figs. 3 and 4. This range is of interest for comparison with the condensation experiments, because these were conducted with condensation rates corresponding to values of $\beta > 10 \times 10^{-3}$, and the liquid–vapour interfacial shear stress was dominated by the mass transfer to the interface.

In Figs. 1 and 2, condensation rates predicted from the modified versions of equations (3) and (5) are shown for different values of ψ. Evidently, condensation experiments with the vertical plate support a value of $\psi = 0{\cdot}75$, while those with the horizontal plate agree better with a value of $\psi = 1{\cdot}0$. Close agreement with the suction experiments is not to be expected for the following two reasons: (a) the suction experiments used uniform suction and hence increasing values of β along the duct, while with condensation, 'suction' was infinite at the beginning of the cooled section and decreased as the liquid film thickened; (b) the suction experiments were carried out in a circular duct, whereas condensation took place only on one wall of a rectangular duct.

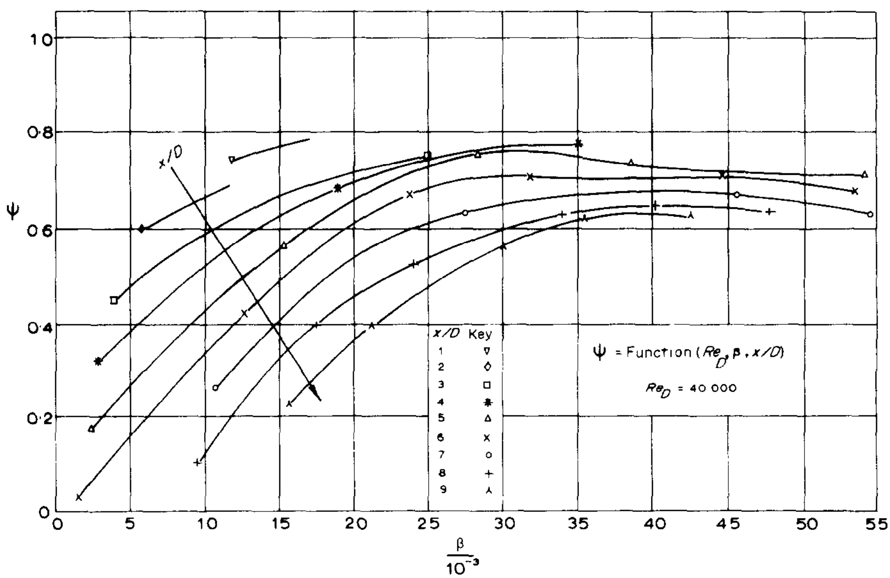

FIG. 4. Local momentum loss coefficient for Porosint tube at $Re_D = 40000$.

The reasonable agreement between the values of ψ obtained from the condensation and suction experiments indicates that the simple division of the shear force into two components—friction drag and momentum—is a valid approach.

5. CONCLUSIONS

The use of a Blasius-type expression for the shear stress at the interface of a turbulently flowing vapour and a laminar condensate film, applied in Nusselt's theory of laminar film condensation, cannot account for measured condensation rates with high vapour velocities.

There is considerable experimental evidence, both from condensation experiments and from experiments with flow in porous tubes, to suggest that condensation rates can be adequately predicted if the shear stress is taken to be equal to the product of the momentum loss coefficient ψ, the mean vapour velocity and the condensation rate per unit area. Although ψ varies with Reynolds number, condensation rate and x/D, a value of 0·75 is probably sufficiently accurate for most design calculations.

It has often been suggested that in counterflow, condensation rates would be lower than those occurring with zero vapour velocity because of the resulting thickening of the condensate film. This is not so, because the film becomes turbulent.

REFERENCES

1. Y. R. Mayhew, D. J. Griffiths and J. W. Phillips, Effect of vapour drag on laminar film condensation on a vertical surface, *Proc. Instn Mech. Engrs* **180**, 280–289, 342–343 (1965–66).
2. Y. R. Mayhew, Comments on the paper 'Theoretical study of laminar film condensation of flowing vapour' by I. G. Shekriladze and V. I. Gomelauri, *Int. J. Heat Mass Transfer* **10**, 107–8 (1967).
3. P. Beckett and G. Poots, Laminar film condensation in forced flows, *Q. J. Mech. Appl. Math.* **25**, 125–52 (1972).
4. J. K. Aggarwal, M. A. Hollingsworth and Y. R. Mayhew, Experimental friction factors for turbulent flow with suction in a porous tube, *Int. J. Heat Mass Transfer* **15**, 1585–1602 (1972).
5. J. K. Aggarwal, Skin friction and heat transfer for turbulent flow with suction in porous tubes, Ph.D. Thesis, University of Bristol (1971).
6. A. Porteous, A theory for the prediction of local phase velocity, static pressure and amount condensed for condensing annular flow in tubes. *Proc. Instn Mech. Engrs* **184**, Part 3G(III), 64–69 (1969–70).

Int. J. Heat Mass Transfer. Vol. 16, pp. 1949–1950 Pergamon Press 1973. Printed in Great Britain

AMBIENT TEMPERATURE STRATIFICATION EFFECTS IN LAMINAR FREE CONVECTION

JACK GOODMAN

Department of Mechanical Engineering, Technion—Israel Institute of Technology, Haifa, Israel

(*Received* 6 *February* 1973)

NOMENCLATURE

a_1, a_2, a_3,	dimensionless constants;
g,	gravitational acceleration;
m, n,	dimensionless constants;
T,	temperature;
x,	distance along plate surface.

Greek symbols

β,	coefficient of thermal expansion;
ν,	kinematic viscosity.

Subscripts

w,	wall value;
∞,	ambient value.